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RESEARCH ARTICLE

A Production Inventory Model with N - Production Rates and Constant Demands

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ABSTRACT

An inventory is nothing but a list of items in stock such as raw materials, work in progress, finished goods, and so on and so forth. At present, a lot of papers on inventory are being published in renowned journals all over the world which indicates the importance of inventory for business organizations. Operations managers think that too little of inventory might hamper their operations; on the other hand, too much of it could ruin the organization. So the wisest thing is to keep it at an optimal quantity level. This paper presents a single product deterministic production inventory model with n (>2) production and constant demand rate. It is assumed that production starts from the inventory level where there is some predetermined quantity as backlogs. For this model, the total cost per unit of time as a function of quantity Q is computed and the results are illustrated by numerical examples. The results show that the cost function is convex.

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1. Introduction

The inventory of goods is essential in every production process. Inventory exists primarily to make goods available to the customers or producers without delay and to increase sales as well as profit. In terms of the character of demand, inventory models are classified into two types. When demand and lead time are known, the model is called deterministic; otherwise, it is called stochastic. In the present paper, the researcher discusses the deterministic inventory model.

The classical inventory model for the single period deterministic demand has been given thought by many researchers like Philip, Ravindran, and Solberg [2001]. Many researchers have dwelt on the inventory model with finite and infinite production rates.

Dave and Choudhauri [1986] talk about the finite rate of production while Silver [1984], Ritchie [1984], Dave [1989], and Urban [1992] pay considerable attention to infinite production rates. Billington [1987]

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considers the classic economic production quantity (EPQ) model without backorders. Bhunia & Maiti [1997] have examined two models: in one model, the production rate is a function of the on-hand inventory, and in another, the same is a function of the demand rate. Ouyang, Chen, and Chang [1999] take into account the continuous inventory system with partial backorders. Rein Nobel and Headen [2000] consider the production inventory model with two discrete production modes. Perumal and Arivarignan [2002] consider a deterministic production inventory model with two different production rates. Cardenas-Barron [2009] has developed an EPQ-type inventory model with planned backorders for determining the economic production quantity for a single product. The product is manufactured in a single-stage manufacturing system that generates imperfect quality products and all these defective products are reworked in the same cycle. Hung-Chi Chang and Chia-Huei Ho [2010] develop an inventory problem for items with imperfect quality and shortage backordering and adopt the renewal reward theorem to derive the expected net profit per unit time and this study applies algebraic methods to derive the exact closed-form solutions for optimal lot size, back ordering quantity, and maximum expected profit. Roy et al. [2011] develop an EOQ model for imperfect items where a portion of demands is partially backlogged. Krishnamoorthy and Panayappan [2013] develop a single-stage production process where defective items produced are reworked and two models of the rework process are considered, an EPQ model without shortage and with shortages. Sivashankari and Panayappan [2014a-d] study and develop several production inventory models of defective items. In their research, they introduce several concepts such as the production inventory model for two levels of production with deteriorating items and shortages, the production inventory model with the reworking of imperfect production, scraps, and shortages, and the production inventory model for two levels of production with defective items and the incorporating multi-delivery policy, the production inventory model for three levels of production with defective items and integrates the cost reduction delivery policy.

Islam et al. [2021] consider the manufacturing process of a single product and a single machine manufactures periodically. They think the defective items can be sent to the recycling process and recycled raw material can be used in the next production cycle to minimize the total cost and the wastage of inventory.

In this paper, the researcher has considered a deterministic production inventory model with n production rates and it is possible that production starts at one rate and after some time it may be switched over to another rate. In this case, the researcher considers the production rates were, $\lambda_1 > \lambda_n$;n ≥ 2 whereas $\lambda_n \geq \lambda_{n-1}$;n ≥ 3. Such a situation is desirable in the sense that since the production is started from a situation where there is some predetermined backlogs quantity, so production rate should be highest for clearing the backlogs as early as possible. Then starting at a lower rate of production a large quantity of stock of manufactured items at the initial stage is avoided, leading to a reduction of the holding cost. Finally, the researcher has established the convexity of the cost function.

Assumptions

i. Production starts when the backlog reaches level $Q_{1.}$
ii. Until meeting the backlogs the production rate λ_1 is highest; the rest of the production rates are $\lambda_2, \lambda_3 .....\lambda_n$ and $\lambda_n \geq \lambda_{n-1}, ...\lambda_2$.
iii. When the backlogs are cleared, the production rates are varied at definite points of time in a cycle.
iv. The production is stopped when the inventory level reaches $Q_n = S$ after which the inventory depletes due to demands alone. On reaching level $-Q_1 (Q_1 \geq 0)$, production starts again.
v. The next switching cost is less than the previous switching cost.

Notations

i. Let t_i be the duration of the production rate λ_i , $i = 1, 2, ..., n$.
ii. K_0 is the initial setup costs.
iii. While changing the production rate there is a switching cost k_i which is while switching from λ_i to λ_{i+1} , $i = 1, 2, ...n-1$.
iv. Let the holding or carrying cost be *h* per unit time and the shortage cost be α per unit.
v. c_i represents production cost per unit when the rate of production is λ_i , $i = 1, 2, ...n$.

2. Model and Analysis

The researcher considers here a deterministic production inventory system with constant demand rate '*a*'. Production starts when the backlog is Q_1 with production rate λ_1, which is considered the highest of all production rates so that backlogs can be cleared as early as possible. At the time t_1, the inventory level reaches 0, and from there the production rate is changed to λ_2 for an amount of time t_2 when the level reaches to Q_2 and so on. When the level finally reaches $Q_n = S$, the production is stopped. From that point the inventory level at the constant rate of '*a*' (see Figure 1).

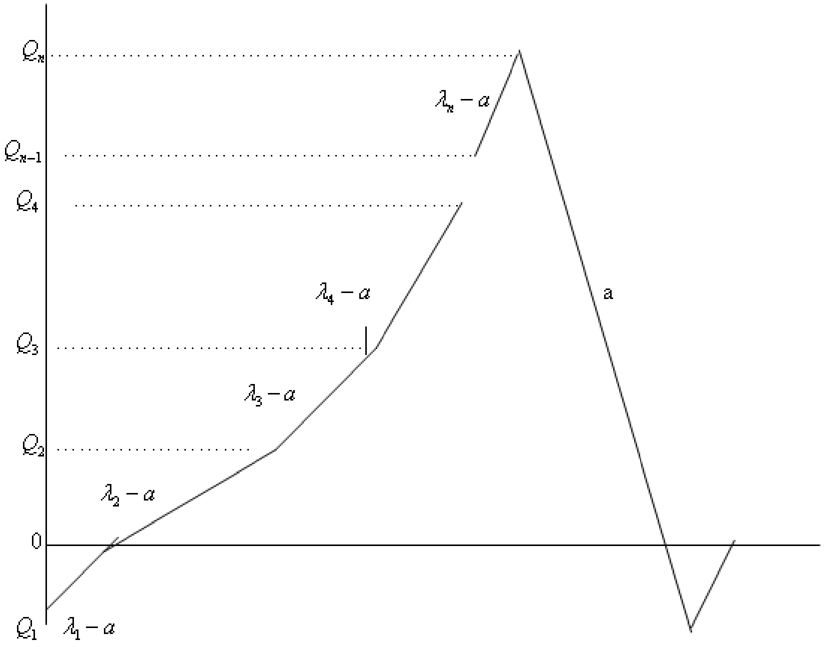

Figure: 1

During 0 to t_1, the inventory is built up at the rate $(\lambda_1 - a)$. The net inventory at time t_1 is zero but $Q_1 = t_1(\lambda_1 - a)$ units are required to yield $t_1 = \dfrac{Q_1}{(\lambda_1 - a)}$. In the same manner we can write t_i 's in terms of λ_i 's & Q_i 's that is,

$$t_2 = \frac{Q_2}{(\lambda_2 - a)}, t_3 = \frac{Q_3 - Q_2}{(\lambda_3 - a)}, t_4 = \frac{Q_4 - Q_3}{(\lambda_4 - a)}, \ldots\ldots\ldots\ldots\ldots, t_n = \frac{Q_n - Q_{n-1}}{(\lambda_n - a)}$$

$$, t_{n+1} = \frac{Q_n}{a}, t_{n+2} = \frac{Q_1}{a}$$

3. Cost Analysis

The researcher has considered a production model in which production rate can be switched from one rate to another. In order to study the effectiveness of this model, the author has decided on the relevant cost function and analyzed it thoroughly.

3.1 Holding cost of the system

The total holding cost of the system is given by

$$= \frac{h}{2}\left\{\sum_{i=2}^{n} \frac{c_i k_i Q_i^2}{(\lambda_i - a)} - \sum_{i=3}^{n-1} \frac{c_i k_i Q_{i-1}^2}{(\lambda_i - a)} + \frac{Q_n^2}{2a}\right\} \qquad (1)$$

3.2 Shortage cost of the system

Total shortage cost of the system is given by

$$= \frac{\alpha Q_1^2}{2}\left(\frac{c_1 k_1}{(\lambda_1 - a)} + \frac{1}{a}\right) \qquad (2)$$

3.3 Total cost function of the system

By using equations (1) and (2) we get the total cost function per unit time as:

$$TC(Q_1,Q_2,......Q_n)=\frac{K_0+\frac{h}{2}\left\{\sum_{i=2}^{n}\frac{c_ik_iQ_i^2}{(\lambda_i-a)}-\sum_{i=3}^{n-1}\frac{c_ik_iQ_{i-1}^2}{(\lambda_i-a)}+\frac{Q_n^2}{2a}\right\}+\frac{\alpha Q_1^2}{2}\left(\frac{c_1k_1}{(\lambda_1-a)}+\frac{1}{a}\right)}{\left(\frac{Q_1+Q_n}{a}+\frac{Q_1}{\lambda_1-a}+\frac{Q_2}{\lambda_2-a}+\sum_{i=3}^{n}\frac{Q_i-Q_{i-1}}{(\lambda_i-a)}\right)} \quad (3)$$

We know that if the product of the Hessian Matrix of a function is greater than or equal to zero, the related function is convex. For proving the convexity of the cost function (3), we shall go for the same. And by induction the researcher has shown that our concerned cost function is convex for all the variables. Generally, to be a convex function the given condition should be satisfied:

$$\left(Q_1,Q_2....Q_n\right)H\left(Q_1,Q_2....Q_n\right)\left(Q_1,Q_2....Q_n\right)^T\geq 0$$

For n=2; we get from (3),

$$TC(Q_1,Q_2)=\frac{K_0+\frac{h}{2}\left\{\frac{c_2k_2Q_2^2}{(\lambda_2-a)}+\frac{Q_2^2}{a}\right\}+\frac{\alpha Q_1^2}{2}\left(\frac{c_1k_1}{(\lambda_1-a)}+\frac{1}{a}\right)}{\left(\frac{Q_1\lambda_1}{(\lambda_1-a)a}+\frac{Q_2\lambda_2}{(\lambda_2-a)a}\right)} \quad (4)$$

Let, $A=\frac{\lambda_1}{(\lambda_1-a)a}$, $B=\frac{\lambda_2}{(\lambda_2-a)a}$, $C=\frac{c_1k_1}{(\lambda_1-a)}+\frac{1}{a}$,

$D=\frac{c_2k_2}{(\lambda_2-a)}+\frac{1}{a}$,

then (4) is reduced to the form

$$TC(Q_1,Q_2)=\frac{K_0+\frac{hDQ_2^2}{2}+\frac{\alpha Q_1^2C}{2}}{\left(AQ_1+BQ_2\right)} \quad (5)$$

Now, differentiating (5) with respect to Q_1 and Q_2 ,we get the relations;

(a) $\dfrac{\partial^2 TC(Q_1,Q_2)}{\partial Q_1^2} = \dfrac{\alpha C}{\left(AQ_1+BQ_2\right)} - \dfrac{2\alpha ACQ}{\left(AQ_1+BQ_2\right)^2} + 2A^2X$

(b) $\dfrac{\partial^2 TC(Q_1,Q_2)}{\partial Q_2^2} = -\dfrac{2BDHQ_2}{\left(AQ_1+BQ_2\right)^2} + \dfrac{DH}{\left(AQ_1+BQ_2\right)} + 2B^2X$

(c) $\dfrac{\partial^2 TC(Q_1,Q_2)}{\partial Q_1 \partial Q_2} = -\dfrac{\alpha BCQ_1 + AhQ_2D}{\left(AQ_1+BQ_2\right)^2} + 2ABX$

(d) $\dfrac{\partial^2 TC(Q_1,Q_2)}{\partial Q_2 \partial Q_1} = -\dfrac{AQ_2hD}{\left(AQ_1+BQ_2\right)^2} - \dfrac{\alpha BQ_1C}{\left(AQ_1+BQ_2\right)^2} + 2ABX$

where $\quad X = \dfrac{K_0 + \dfrac{hDQ_2^2}{2} + \dfrac{\alpha Q_1^2 C}{2}}{\left(AQ_1+BQ_2\right)^3}$

Now, the product of the Hesian Matrix is equal to

$$H_2 = \begin{pmatrix} Q_1 & Q_2 \end{pmatrix} \begin{bmatrix} \dfrac{\partial^2 TC(Q_1,Q_2)}{\partial Q_1^2} & \dfrac{\partial^2 TC(Q_1,Q_2)}{\partial Q_1 \partial Q_2} \\ \dfrac{\partial^2 TC(Q_1,Q_2)}{\partial Q_2 \partial Q_1} & \dfrac{\partial^2 TC(Q_1,Q_2)}{\partial Q_2^2} \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = \frac{2K_0}{\left(AQ_1+BQ_2\right)} \geq 0$$

$$H_2 = \frac{2K_0}{\dfrac{\lambda_1 Q_1}{a(\lambda_1 - a)} + \dfrac{\lambda_2 Q_2}{a(\lambda_2 - a)}} \geq 0 \qquad \mathbf{(6)}$$

As all are positive quantities for n=2 the cost function is convex. Similarly, for n=3 and 4, the product of the Hessian matrices is:

$$H_3 = \frac{2K_0}{\frac{\lambda_1 Q_1}{(\lambda_1 - a)a} + \frac{(\lambda_3 - \lambda_2)Q_2}{(\lambda_2 - a)(\lambda_3 - a)} + \frac{\lambda_3 Q_3}{a(\lambda_3 - a)}} \geq 0 \qquad (7) \quad \text{and}$$

$$H_4 = \frac{2K_0}{\frac{\lambda_1 Q_1}{a(\lambda_1 - a)} + \frac{(\lambda_3 - \lambda_2)Q_2}{(\lambda_3 - a)(\lambda_2 - a)} + \frac{(\lambda_4 - \lambda_3)Q_3}{(\lambda_4 - a)(\lambda_3 - a)} + \frac{\lambda_4 Q_4}{a(\lambda_4 - a)}} \geq 0 \qquad (8)$$

Since all the terms H_3 and H_4 are positive, the corresponding functions are convex. By induction we may write

$$H_n = \frac{2K_0}{\frac{\lambda_1 Q_1}{a(\lambda_1 - a)} + \frac{\lambda_n Q_n}{a(\lambda_n - a)} + \sum_{i=3}^{n} \frac{(\lambda_i - \lambda_{i-1})Q_{i-1}}{(\lambda_i - a)(\lambda_{i-1} - a)}} \geq 0 \qquad (9)$$

Therefore, it can be said that the product of the Hesian matrix for n production rates is positive, indicating that the quantity cost function (3) is convex.

For our convenience, the above equation is valid for any value of n. The researcher has evaluated the optimal value of Q and the corresponding optimal cost function when n takes value 3. For simplicity when n=3, let it be arbitrarily $Q_1 = -Q, Q_2 = Q, Q_3 = 2Q$. Then the cost function (3) is reduced to the form,

$$TC(Q) = \frac{K_0 + \left\{ \frac{hc_2k_2}{2(\lambda_2 - a)} - \frac{hc_3k_3}{2(\lambda_3 - a)} + \frac{2hc_3k_3}{(\lambda_3 - a)} + \frac{2h}{a} \right\} Q^2 + \frac{\alpha Q^2}{2}\left(\frac{c_1k_1}{(\lambda_1 - a)} + \frac{1}{a} \right)}{\left(-\frac{\lambda_1}{(\lambda_1 - a)a} + \frac{\lambda_3 - \lambda_2}{(\lambda_3 - a)(\lambda_2 - a)} + \frac{2\lambda_3}{(\lambda_3 - a)a} \right) Q} \qquad (10)$$

which is the function of Q alone.
In the discrete case, for proving the optimality of Q, we have to show from (10)

$$TC(Q^*) \le TC(Q^* \pm 1)$$

when $TC(Q^*) \le TC(Q^*+1)$ we get

$$\frac{K_0}{\left\{\frac{c_2k_2h}{2(\lambda_2-a)}+\frac{3hc_3k_3}{(\lambda_3-a)}+\frac{c_1k_1\alpha}{2(\lambda_1-a)}+\frac{\alpha}{2a}+\frac{2h}{a}\right\}} \le \left(Q^2+Q\right) \qquad (11)$$

And when; $TC(Q^*) \le TC(Q^*-1)$ we get,

$$\frac{K_0}{\left\{\frac{c_2k_2h}{2(\lambda_2-a)}+\frac{3hc_3k_3}{(\lambda_3-a)}+\frac{c_1k_1\alpha}{2(\lambda_1-a)}+\frac{\alpha}{2a}+\frac{2h}{a}\right\}} \ge \left(Q^2-Q\right) \qquad (12)$$

From equations (11) and (12) combined we get,

$$\left(Q^2-Q\right) \le \frac{K_0}{\left\{\frac{c_2k_2h}{2(\lambda_2-a)}+\frac{3hc_3k_3}{(\lambda_3-a)}+\frac{c_1k_1\alpha}{2(\lambda_1-a)}+\frac{\alpha}{2a}+\frac{2h}{a}\right\}} \le \left(Q^2+Q\right) \qquad (13)$$

So, Q has the optimal value. And this optimal value Q gives the optimum cost of the system

4. Numerical Illustrations

Numerical Illustration: 1

Let

$K_0 = 350,, k_1 = 3.0, k_2 = 3.5, k_3 = 4.0, \lambda_1 = 11.0, \lambda_2 = 9.0, \lambda_3 = 10.0$
$, a = 4.0, c_1 = 1.5, h = 1.0, c_2 = 2.0, c_3 = 2.0, \alpha = 2.0$

Q-Values	Corresponding Cost Function Values
	190.93
6	174.945
7	165.995
8	161.442
$9*$	159.821
10	160.251
11	162.174
12	165.216
13	169.119
14	173.698
15	178.819

From the above the table, it is clear that for the specific value of the parameters the cost function is convex and the optimum value of $Q*$ is 9.

Numerical Illustration: 2

$K_0 = 300,, k_1 = 4.0, k_2 = 4.0, k_3 = 4.0, \lambda_1 = 11.0, \lambda_2 = 9.0, \lambda_3 = 10.0$
$a = 4.0, c_1 = 2.5, h = 1.0, c_2 = 2.0, c_3 = 2.0, \alpha = 2.0$

Q-Values	Corresponding Cost Function Values
2	337.598
3	242.578
4	200.322
5	179.171
6	168.573
7	164.005
$8*$	163.206
9	164.92
10	168.392
11	173.143
12	178.854
13	185.303
14	192.332
15	199.824

From the above the table, it is clear that for the specific value of the parameters the cost function is convex and optimum value of $Q*$ is 8.

5. Conclusion

A number of researchers have considered the single production rate inventory system for making the model simple. But in reality, a single production rate in a production system in particular may affect the advance booking of products which is less realistic. The variation in production rate provides a way resulting in customer satisfaction and earning potential profit by carrying less quantity of inventory and hence minimizing the inventory holding costs [Urban,1999]. In this paper, the researcher has derived the cost function in terms of quantity and shown that the derived cost function is convex. Finally, it is also established that optimal cost function in many areas of production system can provide better results if the production rate is varied rather than uniform or constant. The results of the model could provide the managers with vital information for running the system smoothly.

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Declaration of Interests

I, the authors of this research manuscript, declare that I have no financial interest. I have provided written consent to publish the paper in this journal.

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